A Surface Evolution Approach to Probabilistic Space Carving

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Objective: 3D Photography

We generate a texture-mapped 3D polygonal model computed from multi-view calibrated 2D photographs taken of a visual scene. This 3D model comprises the geometry and color of scene surfaces.

Calibrated 2D photographs

3D polygonal model

Step 1: Probabilistic Space Carving

We first perform probabilistic space carving, which results in a 3D grid of voxel probabilities that describe the likelihood of a voxel existing in the model. This is computed using Bayes' theorem.

$$P(\exists_{klm}=1 \mid D) = \frac{P(D \mid \exists_{klm}=1)P(\exists_{klm}=1)}{P(D \mid \exists_{klm}=1)P(\exists_{klm}=1)+P(D \mid \exists_{klm}=0)P(\exists_{klm}=0)}$$

$\exists_{klm}=1$ Voxel [klm] exists in model

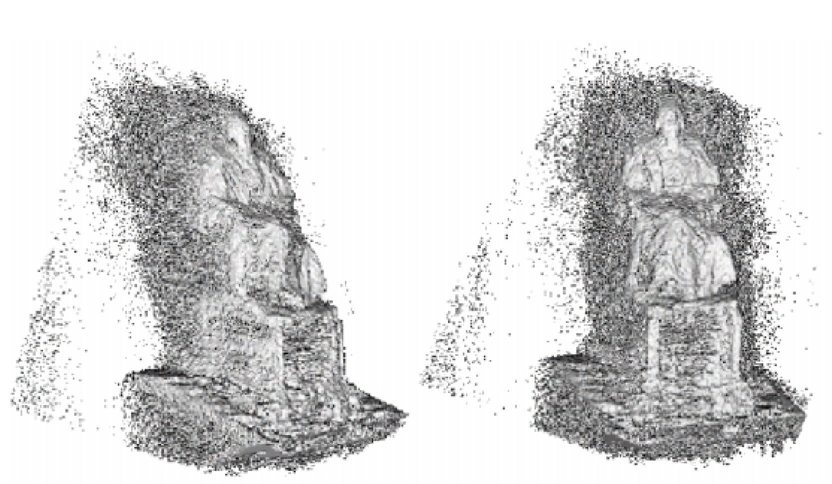
$\exists_{klm}=0$ Voxel [klm] does not exist in model

D Observed data (photographs)

More details in Adrian Broadhurst's Ph.D. thesis, "A probabilistic framework for the Space Carving algorithm", 2001

Initial Surface

For each pixel from each viewpoint, we determine the most likely voxel (if any) along a ray back-projected into the voxel space. These voxels define our initial surface.



Two views of the initial surface.

Step 2: Surface Evolution

We compute a most likely reconstructed surface S that minimizes surface area weighted by a conformal factor Φ.

$$E(S) = \int_S \Phi \, dA \quad \text{where} \quad \Phi = \frac{1}{1+P(\exists_{klm}=1 \mid D)}$$

This is achieved by deforming the initial surface subject to the gradient flow

$$\frac{\partial S}{\partial t} = \Phi H \vec{\mathbf{N}} - (\nabla\Phi \cdot \vec{\mathbf{N}})\vec{\mathbf{N}}$$

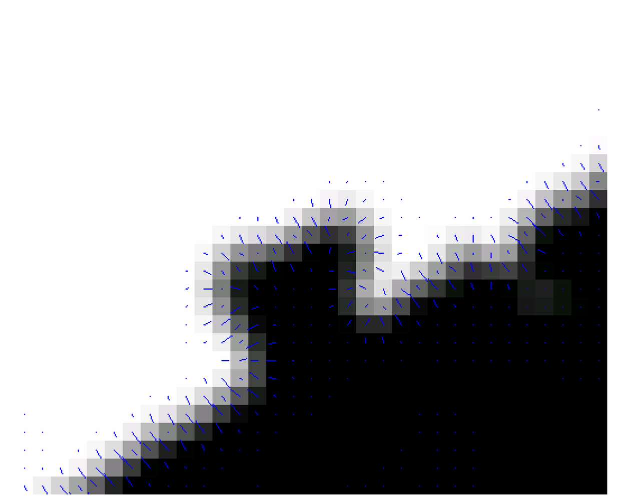
where H is the mean curvature
and $\vec{\mathbf{N}}$ is the inwardly pointing surface normal

Intuition:

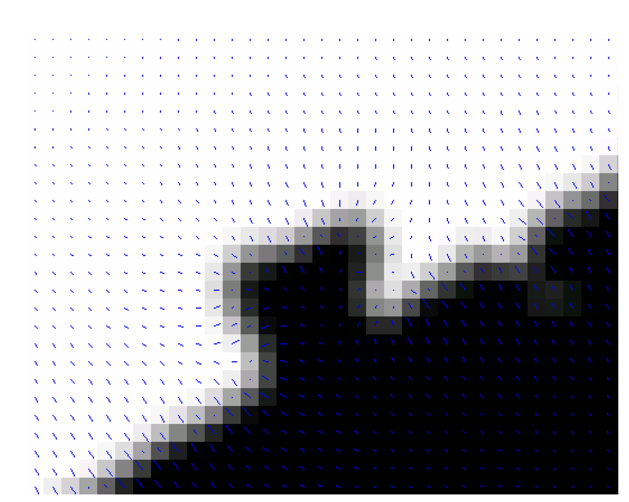
- Low probability - $\Phi \approx 1$ and $\nabla\Phi \approx 0$ Here, the first term in the flow dominates, resulting in smoothing.
- High probability - $\Phi \approx 0$ Here, the second term in the flow dominates, for which the evolving surface locks onto local maxima of the probability.

Diffusion of the Gradient Field

In practice, the gradient field $\nabla\Phi$ has a limited domain of influence. Thus, if the initial surface is not near a local maximum, the gradient term has little influence on the evolution. We extend the gradient field by diffusing it using the PDE approach of Xu and Prince.



Before diffusion

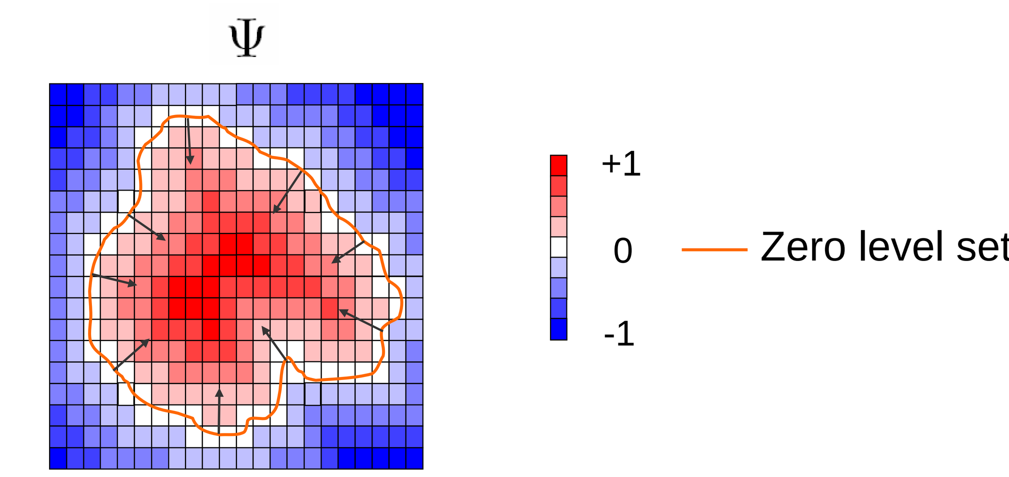


After diffusion

These images plot Φ as a grayscale value, and $\nabla\Phi$ as a needle map for a 2D slice of Φ.

Level Set Methods

Level set methods provide a convenient framework for implementing surface evolution, naturally handling topological changes. The evolving surface is embedded as the zero level set of a function Ψ. At any time, the surface can be extracted from Ψ using the marching cubes algorithm, producing a polygonal model.

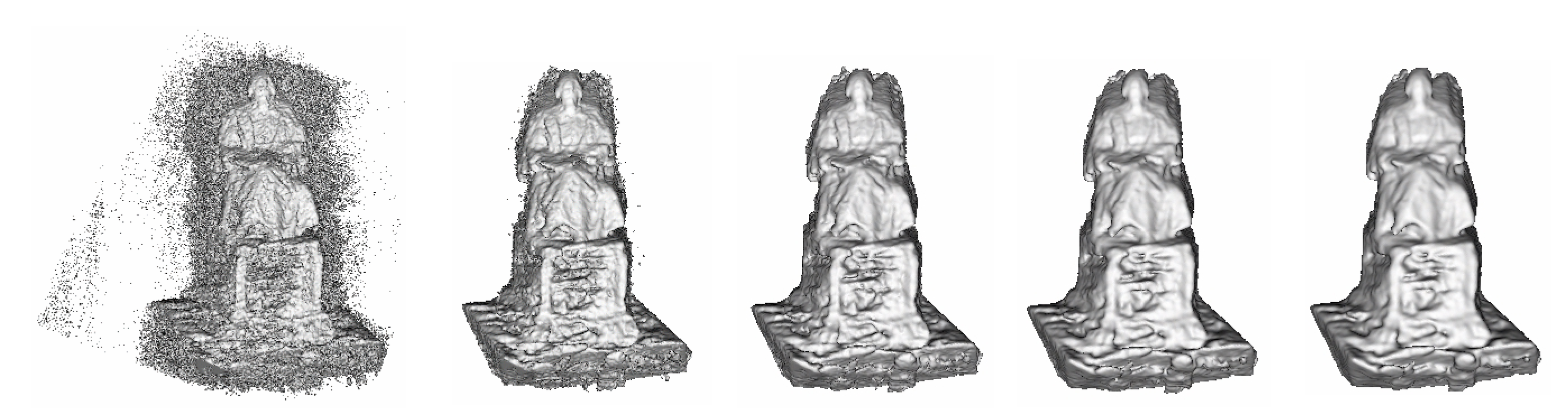


$$\vec{\mathbf{N}} = \frac{\nabla\Psi}{|\nabla\Psi|} \qquad H = \nabla \cdot \vec{\mathbf{N}}$$

Results

Lord Tennyson data set, reconstructed from 13 photographs.

Surface evolution:



Time

New views of the reconstructed 3D polygonal model: